AD-A033 903 ARMY MATERIALS AND MECHANICS RESEARCH CENTER WATERTO--ETC F/G 20/1
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-B--ETC(U)
JUL 76 A G MARTIN

UNCLASSIFIED AMMRC-TR-76-22 NL

1 OF 1
AD A033903

END
DATE
FILMED
2 - 77

AMMRC TR 76-22

(12) B.S.

AD

PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON

ANTHONY G. MARTIN
BALLISTIC MISSILE DEFENSE MATERIALS PROGRAM OFFICE

July 1976

ARMY MATERIALS AND MECHANICS RESEARCH CENTER
Watertown, Massachusetts 02172

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (14) AMMRC-TR-76-22	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER (9)
4. TITLE (and Subtitle) (6) PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON.		5. TYPE OF REPORT & PERIOD COVERED Final Report. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) (10) Anthony G. Martin		8. CONTRACT OR GRANT NUMBER(s) (16)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Army Materials and Mechanics Research Center Watertown, Massachusetts 02172 DRXMR-H		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS D/A Project: 1W162113A661 AMCMS Code: 612113.11.07000 Agency Accession: DA OD4799
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Materiel Development and Readiness Command, Alexandria, Virginia 22333 (11)		12. REPORT DATE July 1976 13. NUMBER OF PAGES 17
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office) (12) 20p.		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Published in the 23rd Defense Conference on Nondestructive Testing, September 1974, San Francisco, NTIAC Proceedings, pp. 150-160

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Composites	Spectrum analysis
Dispersion/Wave	Ultrasonic frequencies
Phase velocity	Velocity measurement

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

(SEE REVERSE SIDE)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 403105

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

LB

Block No. 20

ABSTRACT

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

CONTENTS

INTRODUCTION

A through-transmission method for measuring the phase velocity of ultrasonic waves in dispersive materials is described. The need for such measurement has arisen with the development of composite materials. Since phase velocities of longitudinal and shear waves in solids are functions of elastic properties, velocity measurements shed light on the elastic properties of composites. However, ultrasonic velocities in composites are generally not constant; they vary with frequency. Materials which exhibit such a variation of velocity with frequency are called dispersive materials.

In dispersive materials, a distinction exists between the phase velocity and the group velocity of waves. Phase velocity at a certain frequency can be described as the velocity of a continuous sinusoidal wave of the given frequency. Group velocity is the velocity with which the point of maximum amplitude of a burst (or wave packet) moves through the medium.[1] If phase velocity varies with frequency, the point of maximum amplitude of a burst travels at a velocity different from the velocity of individual cycles in the burst. In such a case an individual cycle will move within the burst as the burst advances. Thus, although measurement of the transit time of bursts is sufficient for group velocity determination, it does not give enough information to enable the calculation of phase velocity in dispersive materials with fixed path length.

This paper describes a method of phase velocity measurement based on the phase between cycles in incident and transmitted bursts. It is an extension of continuous wave methods developed recently for velocity measurements in composites by Lynnworth, Papadakis, and Rea.[2] In their method, a continuous wave (cw) of variable frequency is transmitted through a specimen of fixed length. Beginning in the low ultrasonic range, the frequency is increased gradually. Frequencies at which input and output waves are in phase and out of phase are recorded and the data plotted as in Figure 1, where N is the number of cycles of delay of the signal due to passage through the specimen. At the lowest out-of-phase point, $N = 1/2$. Each successive in-phase and out-of-phase point correspond to an increase of 1/2 cycle ($N = 1/2, 1, 1\text{-}1/2$, etc.). Should the signal be lost because of attenuation but reappear at higher frequencies (such as happens in layered materials), ambiguity exists and additional information is needed to determine N at the higher frequencies.

Since N is the number of cycles of delay of the signal in the specimen, the delay time is N/f and the phase velocity is

$$V = L/(N/f) = Lf/N, \tag{1}$$

where V = phase velocity [m/s],
L = specimen length [m],
f = frequency [Hz].

1. LINDSAY, R. B. *Mechanical Radiation.* McGraw-Hill Book Company, Inc., New York, N. Y., 1960, p. 109-112.
2. LYNNWORTH, L. C., PAPADAKIS, E. P., and REA, W. W. *Ultrasonic Measurement of Phase and Group Velocity and Attenuation Using Continuous Wave Transmission Techniques.* Panametrics, Inc., Contract DAAG46-73-C-0177, Final Report, AMMRC CTR 74-20, April 1974.

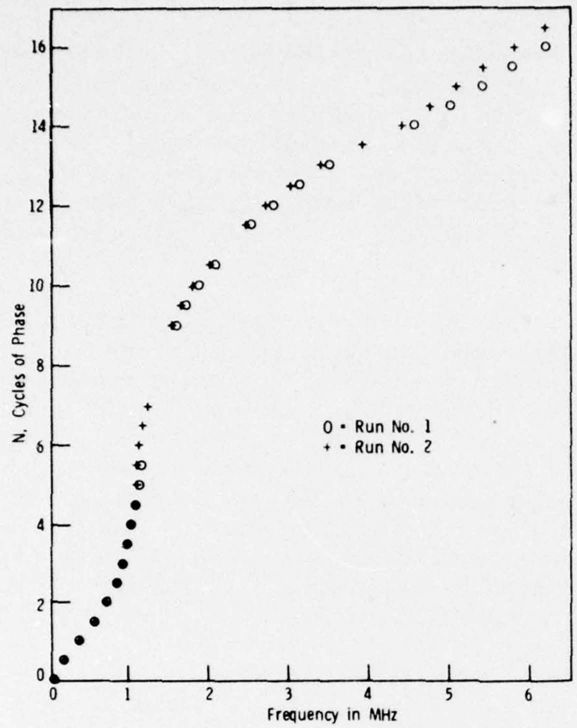

Figure 1. Typical phase data, cycles versus frequency, longitudinal waves

One disadvantage of this method is its lack of discrimination against reflections, mode conversions, or other interfering singals. In the presence of interference, the phase of the transmitted wave may be affected. This disadvantage is overcome while the advantages of phase comparison are retained in the new method in which repetitive narrow-band bursts are used instead of a continuous wave.

Phase comparison between bursts with different delay times requires that a series of bursts be generated in such a way that they are all identical in shape so that they will match exactly on synchronized scope sweeps. The frequency of the carrier must be continuously variable and measurable to enable accurate delay time determination from phase measurement. These objectives are achieved if the bursts are obtained by shaping a cw carrier by pulses derived from a submultiple of the carrier frequency. For dispersive materials, an additional requirement is that the bursts be shaped such as to occupy as narrow a spectrum of frequencies as possible for a given burst width. These requirements are met in the phase comparison method to be described.

ANALYSIS OF PULSES AND BURSTS

As the first step in selecting a pulse with a narrow bandwidth, the spectra of common pulse shapes were compared. Published information[3] indicates that the

3. *Reference Data for Radio Engineers.* International Telephone and Telegraph Corp., New York, N. Y., Fourth Edition, 1956, p. 1019-1024.

spectrum of a cosine-squared pulse has a relatively narrow bandwidth. Rectangular pulses have a wide bandwidth. By means of Fourier analysis and plots obtained from a programmable calculator-plotter, the characteristics of pulses and bursts were studied.

Rectangular Pulse

The amplitude of the harmonics in a train of rectangular pulses of unit amplitude is given by

$$A_n = \frac{\sin \pi n F t_w}{\pi n F t_w} \tag{2}$$

where A_n = relative amplitude of n^{th} harmonic,
$n = 1,2,3,---$
F = pulse repetition frequency [Hz],
t_w = pulse width [s].

The envelope of harmonic amplitudes as a function of the product of harmonic frequency (nF) times pulse width is given in Figure 2.

Cosine-Squared Pulse

A cosine-squared pulse of unit amplitude can be defined by

$$g(t) = \cos^2 \pi t/t_w, \quad (-1/2 \leqq t/t_w \leqq 1/2), \tag{3}$$

where t = time [s].

A train of these pulses is approximated by the series:

$$g(t) \doteq \frac{1}{2m} + \frac{1}{m} \sum_{n=1}^{2m} A_n \cos 2\pi n F t, \tag{4}$$

where $m = 1/(Ft_w)$.

A plot of its envelope of harmonic amplitudes is included in Figure 2. Beyond $nFt_w = 2$, $(n = 2m)$, the amplitudes are less than 2.7% relative to the fundamental. If frequencies beyond $nF = 2/t_w$ are ignored, the cosine-squared pulse is only slightly distorted. By comparison, a rectangular pulse of the same duration would require more than six times the band of frequencies required by the cosine-squared pulse. Other pulse shapes were considered (Gaussian, triangular, damped exponential, etc.) but none was judged better than the cosine-squared pulse.

Narrow-Band Burst

A series of bursts can be generated by 100% amplitude modulation (am) of cw by a repeated pulse. In am the frequency components consist of the unmodulated carrier frequency plus upper and lower sidebands. The sidebands are sums and differences between the carrier frequency and the modulating frequency components. Thus for a narrow-band burst, a modulating function with a narrow bandwidth is needed. The cosine-squared pulse was selected as the modulating function.

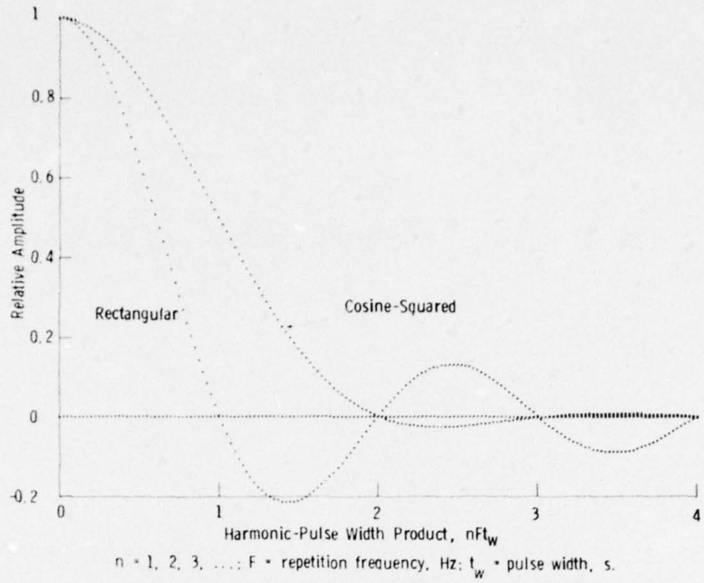

Figure 2. Comparison of harmonic content in square and cosine-squared pulses

A burst with a cosine-squared envelope is represented by

$$g(t) = (\cos^2 \pi t/t_w) \cos 2\pi f_o t, \qquad (-1/2 \leq t/t_w \leq 1/2), \tag{5}$$

where f_o = carrier frequency [Hz].

Let the burst duration be sufficiently long to include several cycles of f_o. Then an approximation to Equation 5 is

$$g(t) \doteq \frac{1}{2m} \sum_{n=-2m}^{2m} A_n \cos 2\pi(f_o + nF)t, \tag{6}$$

where

$$A_n = \begin{cases} 1/2, \text{ where } |n| = m, \\ 1, \text{ where } n = 0, \\ \dfrac{\sin \pi n/m}{\pi(1-n^2/m^2)n/m}, \text{ elsewhere.} \end{cases}$$

Here, n takes on negative as well as positive values. If m is not an integer, the series may be truncated at the integer next larger in magnitude.

The harmonic amplitude envelope is symmetrical about the carrier frequency. The right half has the shape of the envelope of the cosine-squared pulse harmonics. Since $mF = 1/t_w$, the harmonics extend from $(f_o - 2/t_w)$ to $(f_o + 2/t_w)$ and the bandwidth is $4/t_w$ for harmonics with amplitude greater than 2.7% relative to the carrier. This indicates that bandwidth is inversely proportional to pulse width, so that bandwidth is reduced at the expense of pulse width and vice versa. For a given situation, a compromise has to be made between pulse width and bandwidth. For a burst of 20 cycles, the harmonics extend from 10% below to 10% above the carrier frequency.

A cosine-squared burst and its harmonics are illustrated in Figure 3. A carrier frequency of 10 MHz, pulse length of 2 μs, and repetition frequency of 125 kHz have been assumed. The energy in the burst is concentrated mainly in frequencies from 9.5 to 10.5 MHz, where amplitudes relative to the carrier range from 0.5 to 1.

EFFECT OF DISPERSION ON A BURST

Let us assume that a cosine-squared burst is launched into a homogeneous elastic specimen. If velocity is a linear function of frequency, the burst leaving the specimen can be represented by substituting $[t-L/(V_o+nFV')]$ for t. Equation 6 becomes

$$g(t,L) \doteq \frac{1}{2m} \sum_{n=-2m}^{2m} A_n \cos \left[2\pi(f_o+nF)\left(t-\frac{L}{V_o+nFV'}\right)\right], \quad (7)$$

where L = specimen length [m],
V_o = phase velocity at f_o [m/s],
V' = dV/df [m].

If the burst width is more than twice the transit time of the bursts (see Appendix), Equation 7 is approximated by

$$g(t,L) \doteq \left[\frac{1}{2m} \sum_{n=-2m}^{2m} A_n \cos 2\pi nF(t-L/U_o)\right] \cos 2\pi f_o(t-L/V_o), \quad (8)$$

where $U_o = V_o/(1-f_oV'/V_o)$ [m/s].

The expression in brackets is the modulating function for the carrier. U_o is the velocity of the envelope at f_o and is called the group velocity. Unless $V'=0$, the envelope and the oscillations travel at different velocities: the envelope at group velocity and oscillations at phase velocity.

Equation 7 can be expressed in units of megahertz for frequency and microseconds for time. It is plotted in Figure 4 for three different linear velocity-frequency relations with a burst of 20 cycles, a carrier frequency of 10 MHz, and V_o = 5,000 m/s. Three cases are illustrated: $V' = 0$, $+ 0.25\text{x}10^{-3}$m, and $- 0.25\text{x}10^{-3}$m. As calculated, U_o = 5,000 m/s, 10,000 m/s, and 3,333 m/s, respectively. For an assumed 0.01-m thickness, the center of the burst emerges at 2, 1, or 3 μs after it is launched into the specimen. The examples discussed show that the group velocity has varied from 2/3 to twice the phase velocity of the carrier. It is evident that transit time measurement is not sufficient to determine phase velocity in dispersive materials.

In all three cases, cycles near the center of the bursts are exactly or very nearly in phase with continuous waves after travel under the same conditions. The departure from in-phase condition is equivalent to less than 0.1% of the transit time of the bursts. Criteria for monitoring and controlling error due to dispersion are discussed in the Appendix. On the basis of the analysis of these bursts, it was decided to use them for phase velocity determination by phase measurements.

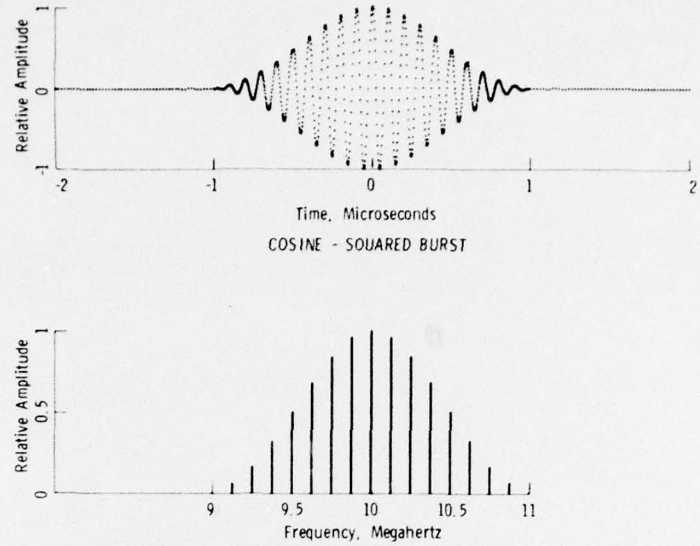

Figure 3. (top) Cosine-squared burst
(bottom) Frequency components of cosine-squared burst

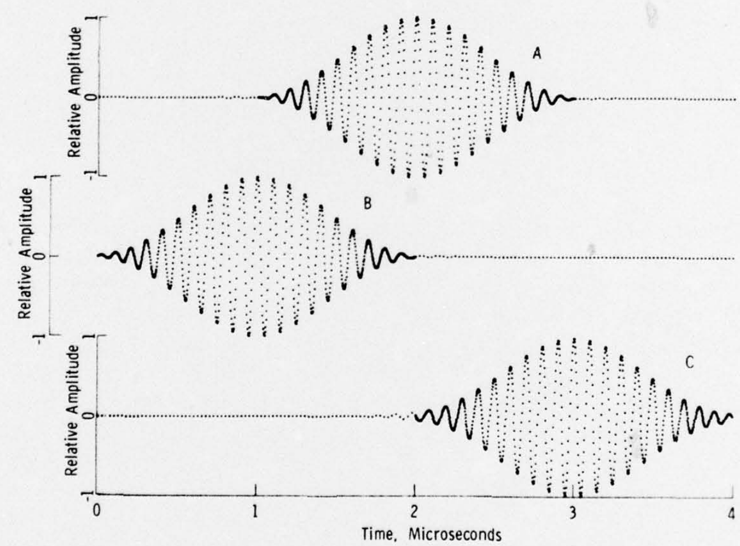

Figure 4. (a) Burst transmitted through medium in which velocity is constant
(b) Burst advanced by increase of velocity with frequency
(c) Burst retarded by decrease of velocity with frequency

PHASE-COMPARISON SYSTEM

Equipment for the narrow-band burst phase comparison system is shown photographically in Figure 5 and in block form in Figure 6. It is comprised of oscillator and modulator A, frequency divider B, frequency counter C, waveform generator D, transducer and specimen assemblies E, preamplifiers F, and oscilloscope G.

The oscillator and modulator are combined into an amplitude-modulated signal generator. It generates continuous sine waves which can be varied from audio frequencies to 11 MHz and has a modulation input for amplitude modulation by an external voltage to produce a 15 V peak-to-peak output into 50 ohms. A square wave output synchronous with the continuous sine wave is also provided. It is connected to the input of frequency divider B.

Frequency divider B was assembled from digital modules. It produces a square wave at a frequency 1/1024 times the input frequency. The output is fed to waveform generator D.

Frequency counter C is a digital counter which measures the output frequency of B. Waveform generator D is triggered by the frequency divider. For every square pulse from B, a single cycle of a cosine-squared wave is generated. The output is fed into the modulation input of the modulator.

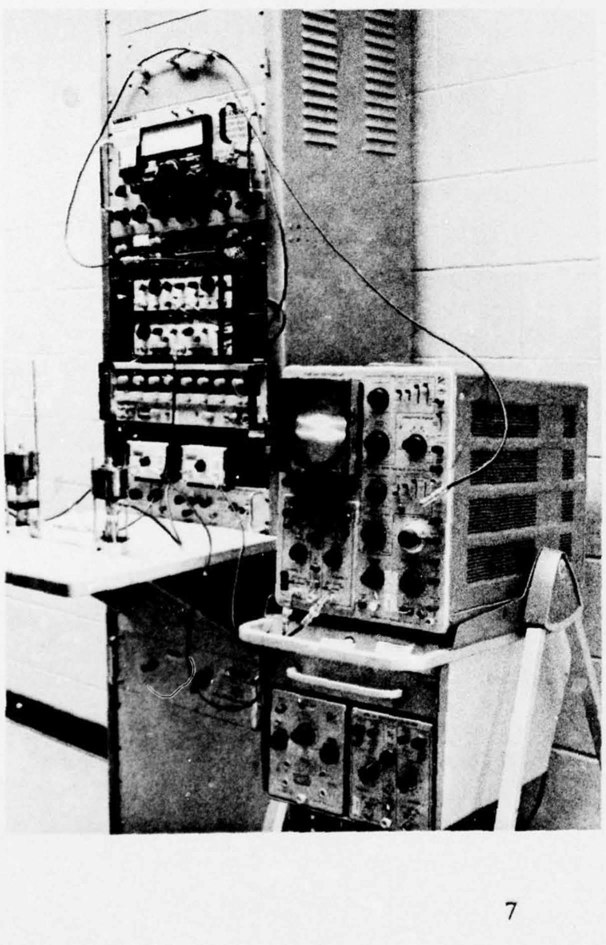

Figure 5. Ultrasonic equipment

19-066-440/AMC-74

After proper adjustment of frequency, dc level, and amplitude in D, the output of A consists of ultrasonic bursts of several cycles with a cosine-squared envelope, repeated at regular intervals. The frequency of vibrations is 1024 times the frequency measured by C. All the bursts are identical, since they are triggered at an integral submultiple of the carrier frequency.

Examples of wave forms at different outputs are given in Figure 7. First, an unmodulated carrier wave obtained from the oscillator section of A is illustrated in Figure 7a. Next, in Figure 7b, is the output from D, the cosine-squared pulse used as the modulating signal. The output of the modulator section of A is the cosine-squared burst illustrated in Figure 7c.

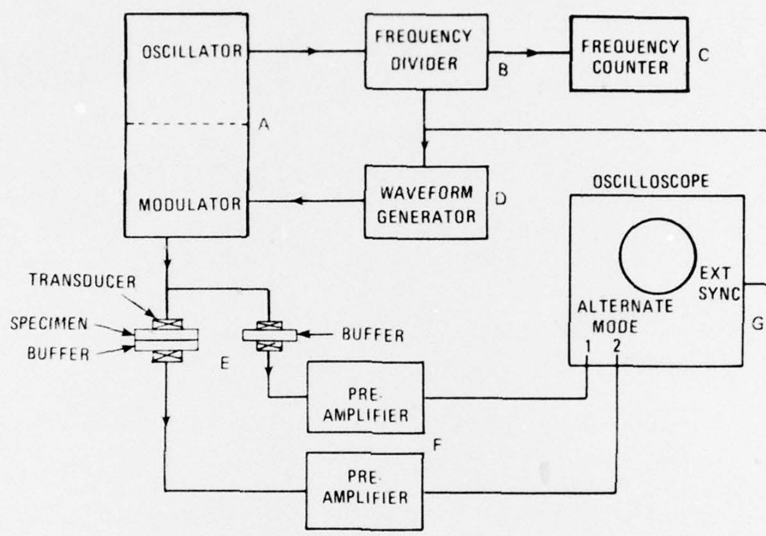

Figure 6. Narrow-band burst phase comparison apparatus

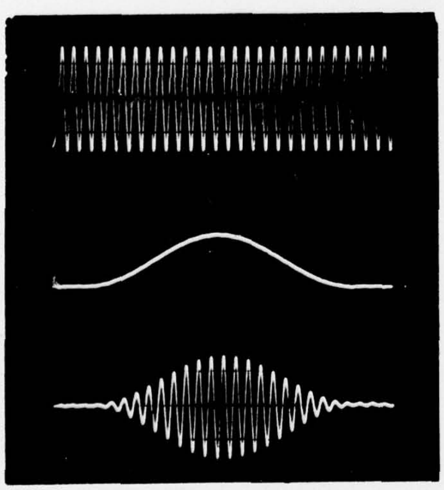

Figure 7. (a) Unmodulated carrier wave

(b) Cosine-squared pulse

(c) Modulated carrier wave

The burst is applied to two identical wideband transducers: 1.9-cm diameter, 5-MHz center frequency for longitudinal waves; 1.27-cm diameter, 2.25-MHz center frequency for shear waves. The specimen is coupled to one of these. Although optimum size requirements have not been determined, specimens measuring 5 cm x 5 cm x 1.27 cm are satisfactory. Coupled to the specimen and to the other transducer are two fused silica buffers of equal thickness (within 0.001 cm). These are working grade optical flats 1.573-cm thick, 5 cm in diameter (replacing 2.5-cm diameter buffers shown in Figure 5). Their purpose is to delay ultrasonic signals, thus avoiding spurious signals caused by direct electrical transfer of signals between transmitting and receiving transducers or circuits. Two more transducers, identical to the first pair, are used as receivers in contact with the buffers.

The received signals are amplified in two wideband amplifiers F and applied to two channels of oscilloscope G which is synchronized by the output of frequency divider B.

PROCEDURE

A preliminary check is made with the two sets of transducers and buffers without a specimen to assure that the received bursts are in phase. Glycerine is used as a couplant for longitudinal waves, DC275-V9 for shear waves. Shear transducer pairs are oriented to the same plane and polarity of vibration. Now the specimen is inserted between one transducer and its buffer.

The carrier frequency is lowered to audio frequencies and the amplitudes of the images from the buffer alone and from the buffer and the specimen are equalized by adjusting the gain. Then the burst duration is adjusted to more than twice the transit time; that is, the width of the bursts is made longer than twice the time between maximum points on the bursts transmitted through the buffer alone and through the buffer-specimen set. Generally, 20 or more cycles are included. Cycles midway between the two images will have about equal amplitudes which are larger than one-half but smaller than full amplitude.

As frequency is increased, N increases by 1/2 for each successive in-phase and out-of-phase condition. Data are plotted as cycles N versus frequency f. If no points are missed, the plotted curve extrapolates to N=0 at f=0 and is continuous until the signal is lost by attenuation. The phase velocity at each point is obtained by use of Equation 1.

As an example, a burst transmitted through a buffer is shown in Figure 8a. After transmission through an identical buffer and a specimen and with additional gain, the burst is as in Figure 8b. For clarity, the oscillogram was taken at a carrier frequency of 3.49 MHz (a frequency which passed through the specimen with low distortion). Figure 9 illustrates the images superposed to enable phase comparison. Here, phase and group velocities are equal and the burst is delayed three cycles in passing through the specimen (N=3, as determined by measurements at various thicknesses).

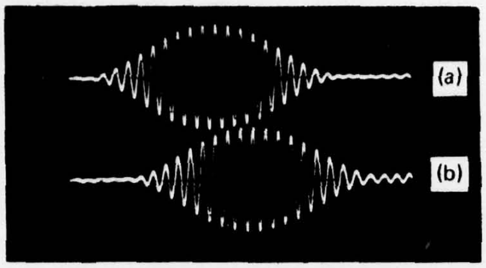

Figure 8. (a) Burst transmitted through buffer alone
(b) Burst transmitted through buffer and specimen

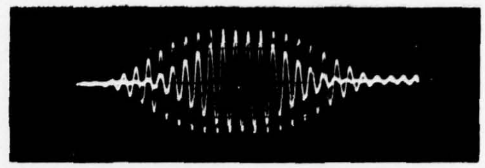

Figure 9. Images superposed for phase comparison

RESULTS

As a test of the method, longitudinal wave measurements were made on a steel rod 0.475 cm in diameter and 18.0 cm in length. The phase velocities obtained along with the curve calculated by Bancroft[3] for a material with Poisson's ratio of 0.30 are plotted in Figure 10.

The ordinate is in terms of the ratio of velocity to the velocity limit approached as the diameter/wavelength ratio approaches zero. For the measured data, the lowest frequency at which a measurement was taken was 114 kHz. At that frequency, the diameter/wavelength ratio is 0.1 and the calculated velocity is 99.8% of the velocity limit approached at low frequency. Accordingly, this velocity was used as the approximate velocity limit for the measured data. The phase velocity varies as expected for diameter/wavelength ratios less than 1.

Data were taken also on a three-dimensional carbon-phenolic composite. The specimen measures 1.278 cm in the z-direction and approximately 5 cm in the x- and y-directions. The center-to-center spacing of fibers in the z-direction is 0.076 cm. Nearly flat planes of fibers parallel to the x-direction alternate with similar planes of fibers parallel to the y-direction. The thickness of a pair of planes which constitutes a unit cell is approximately 0.074 cm. Measurements were taken from low frequency to the point where the signal disappears. The results are given in Figure 11 for velocity in the z-direction.

With either the continuous wave or the burst method, the signal disappears as 1.35 MHz is approached. At higher frequencies, there is uncertainty in the absolute value of N. For that reason, measurements with the burst method are not being reported at the higher frequencies, pending further study.

DISCUSSION

Results on the thin rod agree closely with the theoretical dispersion of extensional waves in rods through most of the frequency range covered. Differences at small diameter/wavelength ratios may be due to experimental error.

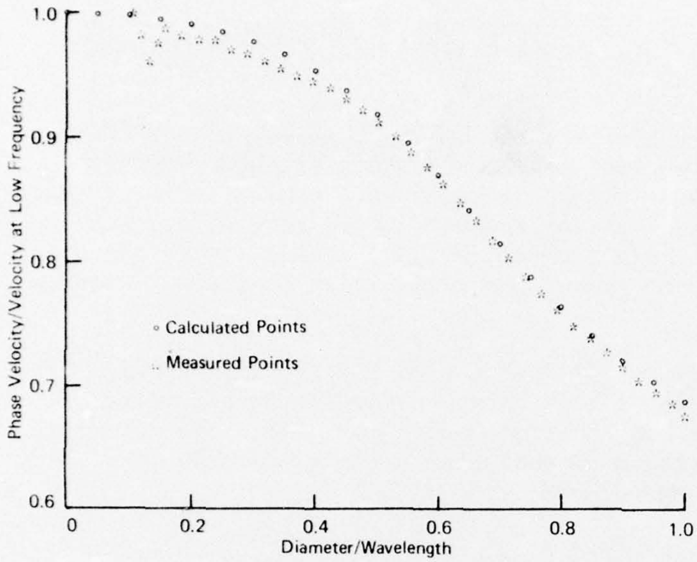

Figure 10. Measured and calculated extensional phase velocity in a steel rod

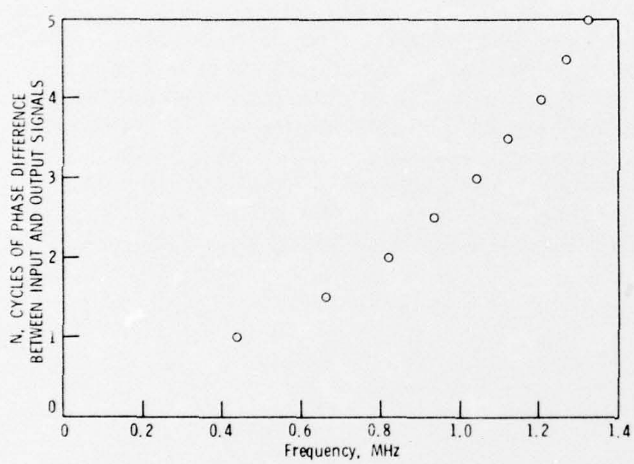

Figure 11. Cycles of phase difference versus frequency in 3-D carbon-phenolic specimen

The results on the composite specimen (Figure 11) are close to the results obtained with continuous waves (Figure 1) up to the point of disappearance of the signal at about 1.4 MHz.

Whether continuous or pulsed waves are used, certain bands of frequencies are highly attenuated in 3-D composites. The center frequencies of the bands vary with the spacing and size of fibers in the three directions. The effect appears related to phenomena associated with layered media[4,5] complicated by the presence

4. ROBINSON, C. W., and LEPPELMEIER, G. W. *Experimental Verification of Dispersion Relations for Layered Composites.* Journal of Applied Mechanics, v. 41, no. 1, Trans. ASME, v. 96, Series E, March 1974, p. 89-91.
5. SUTHERLAND, H. J., and LINGLE, R. *Geometric Dispersion of Acoustic Waves by a Fiber Composite.* J. Composite Materials, v. 6, October 1972, p. 490-502.

of fibers parallel to the direction of wave propagation. At frequencies above the first attenuated band, the phase comparison method becomes ambiguous since the number of cycles of delay is unknown. This difficulty is inherent in any method based on fixed specimen thickness.

It appears possible to eliminate ambiguity by taking observations at fixed frequencies after small changes of specimen thickness. Work is under way to determine phase change versus thickness change at selected frequencies after phase measurements at constant thickness and varying frequency have been made. With very small thickness changes, phase changes should be unambiguous. With the additional data, it should be possible to remove the ambiguities that may accompany the phase-frequency data.

If it is desired, group velocity is obtainable from $U \doteq L\overline{\Delta f}/\overline{\Delta N}$, where $\overline{\Delta f}$ is the frequency increment which accompanies the change in the number of cycles of phase, $\overline{\Delta N}$. Provided that within the band of frequencies included in $\overline{\Delta f}$ the signal is not lost by attenuation, then there is no ambiguity since group velocity depends on phase increment rather than on phase.

CONCLUSIONS

A phase comparison method for narrow-band bursts that provides data for phase and group velocity determinations has been described. The measurements are unequivocal as the frequency is increased from low to high ultrasonic frequencies till the signal disappears. Then in common with other methods based on constant thickness specimens, the data become ambiguous at higher frequencies if the signal returns. It appears possible to remove such ambiguity by obtaining data after small thickness changes. The new method will facilitate ultrasonic studies of wave propagation and related properties in composites and other dispersive materials.

APPENDIX. ANALYSIS OF ERROR DUE TO PHASE SHIFT

The argument of the cosine function in Equation 7 can be expanded as follows:

$$2\pi(f_o+nF)[t-L/(V_o+nFV')] = 2\pi f_o(t-L/V_o) + u_n, \qquad \text{(A-1)}$$

where $u_n = 2\pi nF\left[t - \dfrac{L}{U_o(1+nFV'/V_o)}\right]$,

$U_o = V_o/(1-f_oV'/V_o)$ [m/s].

U_o is the group velocity, measured from the point of maximum amplitude.[1]

In the following discussion, the "o" subscript is omitted from the variables dependent on frequency. They are understood to be at the carrier frequency. Now, u_n can be expressed in terms of known or observable parameters in ten steps:

a. $F = 1/(mt_w)$.

b. Let R = group transit time/burst duration = $(L/U)/t_w$,

c. Let M = number of cycles between peaks of transmitted and received bursts = fL/U.

d. The number of cycles of delay, $N = L/\lambda$.

e. From the definition of group velocity, $U = d\omega/dk = d(2\pi f)/d(2\pi/\lambda) = Ldf/dN$.

f. $fdN/df = fL/U = M$.

g. $V = Lf/N$.

h. $V' = dV/df = \dfrac{L}{N}\left(1 - \dfrac{f}{N}\dfrac{dN}{df}\right) = (L/N)(1-M/N)$.

i. $V'/V = (1-M/N)/f$.

j. Finally, $u_n = 2\pi R\dfrac{n}{m}\left[\dfrac{tU}{L} - \dfrac{1}{1+R\frac{n}{m}(\frac{1}{M} - \frac{1}{N})}\right]$.

Equation 7 now can be expanded trigonometrically to

$$g(t,L) \doteq \frac{1}{2m}\left\{\left[\sum_{n=-2m}^{2m} A_n \cos u_n\right] \cos 2\pi f_o(t-L/V_o) \right. \tag{A-2}$$

$$\left. -\left[\sum_{n=-2m}^{2m} A_n \sin u_n\right] \sin 2\pi f_o(t-L/V_o)\right\} .$$

This is equivalent to two waves 90° out of phase, modulated by the functions in brackets. At small values of u_n, the major component is the cosine wave. The result of summing the components is equivalent to reducing the amplitude of the sine wave slightly and shifting it through a small phase angle:

$$\phi = \text{arc tan}\left\{\frac{\sum_{n=-2m}^{2m} A_n \sin u_n}{\sum_{n=-2m}^{2m} A_n \cos u_n}\right\} \tag{A-3}$$

The accuracy of velocity measurement depends on the phase angle. During a test, the burst duration is adjusted so that it is more than twice the time between maximum points of the images from the two receiving transducers. Consequently, $R<1/2$. Next, the amplitudes of the bursts are equalized by adjusting amplifier gain. At the point where phases are compared (midway between image peaks), $tU/L=1/2$. Now if $|1/M-1/N|<0.07$, the error introduced by phase shift is less than 1% of the duration of one cycle of f_o. This can be monitored readily while taking measurements and controlled by adjusting pulse width. For example, if $|1/M-1/N|=0.25$, the error can be kept to 1% of the duration of one cycle by adjusting the burst duration to three times the transit time. Under these conditions, Equations 7 and 10 are approximated by Equation 8.

DISTRIBUTION LIST

No. of Copies	To
	Office of the Director, Defense Research and Engineering, The Pentagon, Washington, D. C. 20301
1	ATTN: Mr. J. Persh, Staff Specialist for Materials and Structures
12	Commander, Defense Documentation Center, Cameron Station, Building 5, 5010 Duke Street, Alexandria, Virginia 22314
1	Metals and Ceramics Information Center, Battelle Memorial Institute, 505 King Avenue, Columbus, Ohio 43201
	Chief of Research and Development, Department of the Army, Washington, D. C. 20310
2	ATTN: Physical and Engineering Sciences Division
	Commander, Army Research Office, P. O. Box 12211, Research Triangle Park, North Carolina 27709
1	ATTN: Information Processing Office
	Commander, U. S. Army Materiel Development and Readiness Command, 5001 Eisenhower Avenue, Alexandria, Virginia 22333
1	ATTN: DRCDE-TC
1	DRCDE-TC, Dr. R. Zentner
	Commander, U. S. Army Electronics Command, Fort Monmouth, New Jersey 07703
1	ATTN: DRSEL-G-DD
1	DRSEL-GG-DM
	Commander, U. S. Army Missile Command, Redstone Arsenal, Alabama 35809
1	ATTN: Technical Library
1	DRSMI-RSM, Mr. E. J. Wheelahan
	Commander, U. S. Army Armament Command, Rock Island, Illinois 61201
2	ATTN: Technical Library
	Commander, White Sands Missile Range, New Mexico 88002
1	ATTN: STEWS-WS-VT
	Commander, Aberdeen Proving Ground, Maryland 21005
1	ATTN: STEAP-TL, Bldg. 305
	Commander, Frankford Arsenal, Philadelphia, Pennsylvania 19137
1	ATTN: Library, H1300, Bl. 51-2
1	SARFA-L300, Mr. J. Corrie
	Commander, Harry Diamond Laboratories, 2800 Powder Mill Road, Adelphi, Maryland 20783
1	ATTN: Technical Information Office
1	AMXDO-RBF, Dr. R. Oswald
	Commander, Picatinny Arsenal, Dover, New Jersey 07801
1	ATTN: SARPA-RT-S
1	SARPA-FR-M-D, PLASTEC, A. M. Anzalone
	Commander, Redstone Scientific Information Center, U. S. Army Missile Command, Redstone Arsenal, Alabama 35809
4	ATTN: DRSMI-RBLD, Document Section
	Commander, U. S. Army Aeromedical Research Unit, P. O. Box 577, Fort Rucker, Alabama 36460
1	ATTN: Technical Library
	Director, Eustis Directorate, U. S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia 23604
1	ATTN: Mr. J. Robinson, SAVDL-EU-SS
	Commander, USACDC Air Defense Agency, Fort Bliss, Texas 79916
1	ATTN: Technical Library
	Commander, U. S. Army Engineer Waterways Experiment Station, Vicksburg, Mississippi 39180
1	ATTN: Research Center Library

No. of Copies	To
	Technical Director, Human Engineering Laboratories, Aberdeen Proving Ground, Maryland 21005
1	ATTN: Technical Reports Office
	Naval Research laboratory, Washington, D. C. 20375
1	ATTN: Dr. J. M. Krafft - Code 8430
	Chief of Naval Research, Arlington, Virginia 22217
1	ATTN: Code 471
	Naval Surface Weapons Center, Silver Springs, Maryland 20910
1	ATTN: Mr. L. F. Gowen
1	Dr. W. C. Lyons
1	Dr. F. P. Stecker
	Naval Ordnance Systems Commander, Department of the Navy, Washington, D. C. 20360
1	ATTN: ORD-03331, Mr. M. Kinna
	Naval Air Development Center, Warminster, Pennsylvania 18974
1	ATTN: Gwynn McConnell
	Air Force Materials Laboratory, Wright-Patterson Air Force Base, Ohio 45433
2	ATTN: AFML/MXE/E. Morrissey
1	AFML/LC
1	AFML/LLP/D. M. Forney, Jr.
1	AFML/MBC/Mr. Stanley Schulman
1	Dr. D. Schmidt
1	Dr. N. J. Pagano
1	Mr. T. Cooper
1	Mr. R. Rawand
	Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico 87117
1	ATTN: Capt. J. Meiggs
1	Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio 45433
	National Aeronautics and Space Administration, Washington, D. C. 20546
1	ATTN: Mr. B. G. Achhammer
1	Mr. G. C. Deutsch - Code RR-1
	National Aeronautics and Space Administration, Marshall Space Flight Center, Huntsville, Alabama 35812
1	ATTN: R-P&VE-M, R. J. Schwinghamer
1	S&E-ME-MM, Mr. W. A. Wilson, Building 4720
	National Aeronautics and Space Administration, Lewis Research Center, 21000 Brookpark Road, Cleveland, Ohio 44135
1	ATTN: Mr. R. H. Kemp
	Panametrics, 221 Crescent Street, Waltham, Massachusetts 02154
1	ATTN: L. Lynnworth
	Ballistic Missile Defense Program Office, Commonwealth Building, 1320 Wilson Boulevard, Arlington, Virginia 22209
1	ATTN: DACS-BMT
	Commander, Ballistic Missile Defense Systems Command, P. O. Box 1500, Huntsville, Alabama 35807
1	ATTN: BMDSC-TEN, Mr. N. Hurst
	Ballistic Missile Advanced Technology Center, P. O. Box 1500, Huntsville, Alabama 35807
1	ATTN: ATC-M, Mr. M. Whitfield
1	ATC-M, Dr. D. Harmon
1	ATC-X, Mr. W. Davis
1	Director, Advanced Research Project Agency, Architect Building, 1400 Wilson Boulevard, Arlington, Virginia 22209

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE
MATERIALS BY NARROW-BAND BURST PHASE
COMPARISON - Anthony G. Martin

Technical Report AMMRC TR 76-22, July 1976, 17 pp -
illus, D/A Project 1W162113A661,
AMCMS Code 612113.11.07000

AD_______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE
MATERIALS BY NARROW-BAND BURST PHASE
COMPARISON - Anthony G. Martin

Technical Report AMMRC TR 76-22, July 1976, 17 pp -
illus, D/A Project 1W162113A661,
AMCMS Code 612113.11.07000

AD_______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE
MATERIALS BY NARROW-BAND BURST PHASE
COMPARISON - Anthony G. Martin

Technical Report AMMRC TR 76-22, July 1976, 17 pp -
illus, D/A Project 1W162113A661,
AMCMS Code 612113.11.07000

AD_______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE
MATERIALS BY NARROW-BAND BURST PHASE
COMPARISON - Anthony G. Martin

Technical Report AMMRC TR 76-22, July 1976, 17 pp -
illus, D/A Project 1W162113A661,
AMCMS Code 612113.11.07000

AD_______________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON - Anthony G. Martin

AD________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

Technical Report AMMRC TR 76-22, July 1976, 17 pp - illus, D/A Project 1W162113A661, AMCMS Code 612113.11.07000

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON - Anthony G. Martin

AD________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

Technical Report AMMRC TR 76-22, July 1976, 17 pp - illus, D/A Project 1W162113A661, AMCMS Code 612113.11.07000

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON - Anthony G. Martin

AD________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

Technical Report AMMRC TR 76-22, July 1976, 17 pp - illus, D/A Project 1W162113A661, AMCMS Code 612113.11.07000

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.

Army Materials and Mechanics Research Center,
Watertown, Massachusetts 02172
PHASE VELOCITY MEASUREMENTS IN DISPERSIVE MATERIALS BY NARROW-BAND BURST PHASE COMPARISON - Anthony G. Martin

AD________________

UNCLASSIFIED
UNLIMITED DISTRIBUTION

Key Words

Composites
Dispersion/wave
Phase velocity

Technical Report AMMRC TR 76-22, July 1976, 17 pp - illus, D/A Project 1W162113A661, AMCMS Code 612113.11.07000

A new method for the measurement of phase velocity of ultrasonic waves by phase comparison of narrow-band bursts is described. The method was developed for the determination of properties such as elastic moduli of composites or other materials in which phase velocity varies with frequency. In such dispersive materials, individual cycles within a burst travel at a velocity different than for the burst as a whole. The new method is based on the phase comparison of entering and emerging sinusoidally-shaped bursts. Two wideband transducers and a buffer are used for reference. The specimen is interposed between a set of transducers and buffer identical with the reference set. Bursts are fed to the two assemblages and the phases of the signals after transmission are compared. The signal is varied from low to high ultrasonic frequencies while in-phase and out-of-phase conditions and frequencies are recorded. From the data, phase velocity is calculated at each recorded frequency. The method and some results are discussed.